FACTORISATION OF QUADRATIC EXPRESSIONS 16/06/20 TUE

LINEAR AND QUADRATIC EXPRESSIONS

Linear expression: this is an expression in which the highest power (or index) of the unknown is 1. Eg $x-1$, $2x+1$ and $2-4x$.

Quadratic expression; this is an expression in which the highest power (or index) of the unknown is two. Eg x^2+2, $2x^2+1$ and $4-x^2$

EXPANSION OF ALGEBRAAIC EXPRESSIONS

The expression $(x+2)(x-5)$ means$(x+2)\times(x-5)$. The product of the two binomials $(x+2)$ and $(x-5)$ is found by multiplying each term in the first binomial by each term in the second binomial.

EXAMPLE

Find the product of $(x+2)$ and $(x-5)$.

SOLUTION

$$(x+2)(x-5)=x(x-5)+2(x-5)$$

$$=x^2-5x+2x-10=x^2+2x-5x-10$$

$$=x^2-3x-10$$

Hence, $(x+2)(x-5)$ is **expanded** as $x^2-3x-10$.

EXAMPLE

Expand: $(2c-3m)(c-4m)$

SOLUTION

$$(2c-3m)(c-4m)=2c(c-4m)-3m(c-4m)$$

$$=2c^2-8cm-3cm+12m^2$$

$$=2c^2-11cm+12m^2$$

EXAMPLE

Expand**:** $(3a+2)^2=(3a+2)(3a+2)$

$$3a(3a+2)+2(3a+2)$$

$$=9a^2+6a+6a+4$$

$$=9a^2+12a+4$$

COEFFICIENT OF TERMS

The coefficient of an algebraic term is the number that precedes (or comes before) the unknown. For example, in $3x^2$, the coefficient of x^2 is 3

In $-2y$, the coefficient of y is -2

In $\frac{2}{3}d$, the coefficient of d is $\frac{2}{3}$

EXAMPLE

Find the coefficient of x in the expansion of $(x+9)(x+3)$

SOLUTION

$$(x+9)(x+3)=x^2+3x+9x+27$$

$$=x^2+12x+27$$

Hence, the coefficient of x in the expansion of

$(x+9)(x+3)$ is $+12$

EXAMPLE

Find the coefficient of ab in the expansion of

$$(5a+2b)(4a-3b)$$

SOLUTION

$$(5a+2b)(4a-3b)=20a^2-15ab+8ab-6b^2$$

$$20a^2+8ab-15ab-6b^2=20a^2-7ab-6b^2$$

Hence,

The coefficient of ab in $(5a+2b)(4a-3b)$ is -7

EXECISE

1. Expand the following:
a. $(d+3)(d-7)$
b. $(m+4n)^2$
c. $(4m-n)(3m-3n)$
d. $(2c-3d)^2$
2. Find the coefficient of;
a. d in the expansion of $(d-8)(d+3)$
b. u in the expansion of $(4u-5)(2u-7)$
c. ab in the expansion of $(5a+2b)(5a-2b)$

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QUADRATIC EXPRESSIONS

Quadratic expression; this is an expression in which the highest power (or index) of the unknown is two. Eg $x^2 + 2$, $2x^2 + 1$ and $4 - x^2$

Since $(x+2)(x-6) = x^2 - 4x - 12$, then $(x+2)$ and $(x-6)$ are the factors of $x^2 - 4x - 12$.

A quadratic expression may **not** have factors. In arithmetic, 13 is called **prime** since it has only two factors; 1 and itself. Similarly, $x^2 + 2x - 6$ has no factors (other than itself and 1)

To factorize a quadratic expression is to express it as a product of its factors. Thus $x^2 - 4x - 12$ factorizes to become$(x+2)(x-6)$. The example below shows the steps to be followed when factorizing quadratic expressions.

EXAMPLE

Factorize the quadratic expression: $x^2 + 7x + 10$

SOLUTION

The problem is to fill the brackets in the statement

$$x^2 + 7x + 10 = (\quad)(\quad)$$

1st step: x appears in each bracket since their product always gives the first termx^2:

$$(x \quad)(x \quad)$$

2nd step: The product of the last terms in the two brackets must be equal to the last term $+10$. Number pairs which have a product of +10 are:

a. +10 and +1
b. +5 and +2
c. -10 and -1
d. -5 and -2

3rd step: the sum of the last terms in the two brackets must be equal to the middle number, in this case +7.

Adding the number pairs above ,we have

a. $(+10) + (+1) = +11$
b. $(+5) + (+2) = +7$
c. $(-10) + (-1) = -11$
d. $(-5) + (-2) = -7$

Of these, only b gives +7. Thus,

$$x^2 + 7x + 10 = (x+5)(x+2)$$

Note:

1. The answer can be checked by expanding the brackets.
2. The order of the brackets is **not** important. $(x+5)(x+2) = (x+2)(x+5)$

EXAMPLE

Factorize; $d^2 + 11d + 18$

SOLUTION

1st step: $d^2 + 11d + 18 = (d \quad)(d \quad)$

2nd step: find two numbers that their **product** is +18 and their **sum** is the middle number +11. Considering positive factors only, we have

	factors of +18	sum of factors
a.	+1 and +18	+19
b.	+2 and +9	+11
c.	+3 and +6	+9

Of these, only b gives the required result. Thus,

$d^2 + 11d + 18 = (d+2)(d+9)$

The above method is called **trial and improvement method**

EXAMPLE

Factorize the following:

1. $t^2 - 10t + 24$
2. $x^2 + 2x - 15$
3. $x^2 - 4x - 12$

SOLUTION

1. 1st step: $t^2 - 10t + 24 = (t \quad)(t \quad)$
   2nd step: find two numbers that their **product** is $+24$ and their sum is -10. Since the 24 is positive and the 10 is negative, consider negative factors only.

factors of +24	sum of factors
a. -1 and -24	-25
b. -2 and -12	-14
c. -3 and -8	-11
d. -4 and -6	-10

Of these, only d gives the required result. Thus,

$$t^2 - 10t + 24 = (t-4)(t-6)$$

2. 1st step: $x^2 + 2x - 15 = (x \quad)(x \quad)$

2nd step: Find two numbers that their **product** is -15 and their sum is +2. Since the 15 Is negative and the 2 is positive, list the possible pairs and their sums.

factors of -15	sum of factors
a. +1 and -15	-14
b. +3 and -5	-2
c. +5 and -3	+2
d. +15 and -1	+14

Of these, only c gives the required result. Thus,

$$x^2 + 2x - 15 = (x+5)(x-3)$$

4. 1st step: $x^2 - 4x - 12 = (x \quad)(x \quad)$

2nd step: Find two numbers that their **product** is -12 and their sum is -4. Since the 12 Is negative and the 4 is negative, list the possible pairs and their sums.

factors of -12	sum of factors
a. -12 and +1	-11
b. -6 and +2	-4
c. -4 and +3	-1
d. -3 and +4	+1
e. -2 and +6	+4
f. -1 and +12	+11

Of these, only b gives the required result. Thus,

$$x^2 - 4x - 12 = (x-6)(x+2)$$

EXERCISE

Factorize the following quadratic expressions:

1. $a^2 + 8a + 15$
2. $b^2 - 8b + 7$
3. $d^2 - 9d + 14$
4. $c^2 - 8c - 20$
5. $f^2 + 6f - 7$

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PERFECT SQUARES

An expression is said to be a perfect square if it can be expressed as a square of another expression.

$$(a+b)^2 = (a+b)(a+b)$$

$$= a^2 + ab + ab + b^2$$

$$= a^2 + 2ab + b^2$$

$$\therefore \boldsymbol{(a+b)^2 = a^2 + 2ab + b^2}$$

Similarly,

$$(a-b)^2 = (a-b)(a-b)$$

$$= a^2 - ab - ab + b^2$$

$$= a^2 - 2ab + b^2$$

$$\therefore \boldsymbol{(a-b)^2 = a^2 - 2ab + b^2}$$

NOTE

- **The square of a sum of two quantities is equal to the sum of their squares plus twice their product.**
- **The square of a difference of two quantities is equal to the difference of their squares minus twice their product.**

EXAMPLE

Expand the following:

a. $(3m + 7n)^2$
b. $(4u - 5v)^2$

SOLUTION

a. $(3m+7n)^2 = (3m)^2 + 2(3m)(7n) + (7n)^2$
$= 9m^2 + 42mn + 49n^2$

b. $(4u-5v)^2 = (4u)^2 - 2(4u)(5v) + (5v)^2$
$= 16u^2 - 40uv + 25v^2$

The expansion of a perfect square can be used to shorten the working when squaring numbers

EXAMPLE

Expand:

a. 104^2

b. 97^2

SOLUTION

a. $104^2 = (100 + 4)^2$
$= 100^2 + 2(100)(4) + 4^2$
$= 10000 + 800 + 16$
$= 10816$

b. $97^2 = (100 - 3)^2$
$= 100^2 - 2(100)(3) + 3^2$
$= 10000 - 600 + 9$
$= 9409$

EXAMPLE

Factorize the following:

a. $h^2 + 12h + 36$
b. $25h^2 - 30hk + 9k^2$

SOLUTION

a. h^2 is the square of h, 36 is the square of 6 and $12h$ is twice the product of 6 and h. Thus,
$$h^2 + 12h + 36 = (h + 6)(h + 6)$$
$$= (h + 6)^2$$
b. $25h^2$ is the square of $5h$, $9k^2$ is the square of $3k$ and $30hk$ is twice the product of $5h$ and $3k$
c. $\therefore 25h^2 - 30hk + 9k^2 = (5h - 3k)^2$

DIFFERENCE OF TWO SQUARES

Consider the expression $(a + b)(a - b)$

$$(a + b)(a - b) = a(a - b) + b(a - b)$$

$$= a^2 - ab + ab - b^2$$

$$= a^2 - b^2$$

$$\therefore \boldsymbol{(a + b)(a - b) = a^2 - b^2}$$

The expression $\boldsymbol{a^2 - b^2}$ is called **difference of two squares.**

Hence,

The difference of the squares of two quantities is equal to the product of their sum and their difference.

EXAMPLE

Simplify each of the following:

a. $(4x + 3y)(4x - 3y)$
b. $(2p + 1)(2p - 1)$

SOLUTION

a. $(4x + 3y)(4x - 3y) = (4x)^2 - (3y)^2$
$= 16x^2 - 9y^2$
b. $(2p + 1)(2p - 1) = (2p)^2 - (1)^2$
$= 4p^2 - 1$

EXAMPLE

Factorize the following:

a. $y^2 - 4$
b. $36 - 9a^2$
c. $25m^2 - 16n^2$
d. $5a^2 - 45$

SOLUTION

a. $y^2 - 4 = (y)^2 - (2)^2$
$= (y + 2)(y - 2)$
b. $36 - 9a^2 = (6)^2 - (3a)^2$
$= (6 + 3a)(6 - 3a)$
c. $25m^2 - 16n^2 = (5m)^2 - (4n)^2$
$= (5m + 4n)(5m - 4n)$
d. $5a^2 - 45 = 5(a^2 - 9)$
$5(a^2 - 3^2)$
$= 5(a + 3)(a - 3)$

EXAMPLE

Using difference of two squares, evaluate:

a. $8.9^2 - 1.1^2$
b. $99^2 - 1$
c. $173^2 - 127^2$

SOLUTION

a. $8.9^2 - 1.1^2 = (8.9 + 1.1)(8.9 - 1.1)$
$= 10 \times 7.8$
$= 78$
b. $99^2 - 1 = (99 + 1)(99 - 1)$
$= 100 \times 98$
$= 9\,800$
c. $173^2 - 127^2 = (173 + 127)(173 - 127)$
$= 300 \times 46$
$= 13\,800$

EXERCISE

1. Expand the following:
a. $(2a + 3d)^2$
b. $(3b - 5c)^2$

2. Find the squares of the following:

a. 103

b. 996

3. Factorize the following:

a. $x^2 + 6xy + 9y^2$

b. $9a^2 - 24ab + 16b^2$

4. Factorize the following:

a. $4m^2 - n^2$

b. $36a^2 - 49b^2$

c. $5c^2 - 45d^2$

5. Find the value of the following:

a. $118^2 - 18^2$

b. $1004^2 - 16$