

FACTORIZATION 09/06/20 TUE

A **number** or **expression** is called a **factor** of another if it divides the other without any remainder. Factorization is the act of representing expressions as a product of its factors.

Removing brackets (revision)

EXAMPLE

Remove brackets from:

- $3(2x - y)$
- $(3a + 8b)5a$
- $-2n(7y - 4z)$

SOLUTION

- $3(2x - y) = 3 \times 2x - 3 \times y$
 $= 6x - 3y$
- $(3a + 8b)5a = 3a \times 5a + 8b \times 5a$
 $= 15a^2 + 40ab$
- $-2n(7y - 4z) = (-2n) \times 7y - (-2n) \times 4z$
 $= 14ny - (-8nz)$
 $= -14ny + 8nz$

COMMON FACTORS

EXAMPLE

Find the HCF of $6xy$ and $18x^2$.

SOLUTION

$$6xy = 6 \times x \times y$$
$$18x^2 = 6 \times 3 \times x \times x$$

\therefore the HCF of $6xy$ and $18x^2$ is $6 \times x = 6x$

COMMON FACTORS OF BINOMIAL EXPRESSIONS

A binomial expression has **two** terms. To factorize an expression is to write it as a product of its **factors**.

EXAMPLE

Factorize the following binomial expressions:

- $9a - 3z$
- $5x^2 + 15x$
- $2mh - 8m^2h$

SOLUTION

- The HCF of $9a$ and $3z$ is 3 .
 $9a - 3z = 3(3a - z)$
- The HCF of $5x^2$ and $15x$ is $5x$
 $5x^2 + 15x = 5x(x + 3)$

- The HCF of $2mh$ and $8m^2h$ is $2mh$.
 $2mh - 8m^2h = 2mh(1 - 4m)$

EXERCISE

- Remove brackets from the following:
 - $5p(9r - 8s)$
 - $-6a(2a - 7b)$
- Find the HCF of the following:
 - $8pq$ and $24p^2$
 - $10ax^2$ and $14a^2x$
- Factorize the following binomial expressions:
 - $5am - 20bm$
 - $3dh + 15dk$

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COMMON FACTORS WITH LARGER EXPRESSIONS

EXAMPLE

Factorize the following:

- $2x(5a + 2) - 3y(5a + 2)$.
- $2d^3 + d^2(3d - 1)$
- $(a + m)(2a - 5m) - (a + m)^2$
- $(x - 2y)(z + 3) - x + 2y$

SOLUTION

- In the given expression, $(5a + 2)$ is common.
Thus

$$2x(5a + 2) - 3y(5a + 2) = (5a + 2)(2x - 3y)$$

- In the given expression, d^2 is common. Thus,

$$2d^3 + d^2(3d - 1) = d^2[2d + (3d - 1)]$$

$$= d^2[2d + 3d - 1]$$

$$= d^2(5d - 1)$$

- In the given expression, $(a + m)$ is common.
Thus,

$$(a + m)(2a - 5m) - (a + m)^2$$
$$= (a + m)[(2a - 5m) - (a + m)]$$

$$= (a + m)[2a - 5m - a - m]$$

$$= (a + m)(a - 6m)$$

- Notice that -1 is a factor of the last two terms.
Thus, the given expression can be written as:

$$(x - 2y)(z + 3) - x + 2y$$
$$= (x - 2y)(z + 3) - 1(x - 2y)$$

In the above expression, $(x - 2y)$ is common.
Thus,

$$\begin{aligned} &= (x - 2y)[(z + 3) - 1] \\ &= (x - 2y)(z + 3 - 1) \\ &= (x - 2y)(z + 2) \end{aligned}$$

SIMPLIFYING CALCULATIONS BY FACTORIZATION

EXAMPLE

By factorizing, simplify: $79 \times 37 + 21 \times 37$

SOLUTION

37 is a common factor of 79×37 and 21×37 .

$$\begin{aligned} 79 \times 37 + 21 \times 37 &= 37(79 + 21) \\ &= 37 \times 100 \\ &= 3700 \end{aligned}$$

EXAMPLE

Factorization the expression: $\pi r^2 + 2\pi r h$. Hence find the value of $\pi r^2 + 2\pi r h$ when $= \frac{22}{7}$, $r = 14$ and $h = 43$.

SOLUTION

$$\pi r^2 + 2\pi r h = \pi r(r + 2h)$$

When $\pi = \frac{22}{7}$, $r = 14$ and $h = 43$

$$\begin{aligned} \pi r^2 + 2\pi r h &= \pi r(r + 2h) \\ &= \frac{22}{7} \times 14(14 + 2 \times 43) \\ &= 22 \times 2(14 + 86) \\ &= 44 \times 100 \\ &= 4400 \end{aligned}$$

EXERCISE

- Factorize the following:
 - $a^2(5a - 3b) - 3a^3$
 - $(2u - 3v)(3m - 4n) - (2u - 3v)(m + 2n)$
 - $(2a - 3b)(c + d) - 2a + 3b$
- By factorizing, simplify the following:
 - $\frac{8}{13} \times 125 + \frac{5}{13} \times 125$
 - $762 \times 87 - 562 \times 87$
- Factorize the expression:
 - $\pi r^2 h + \frac{1}{3} \pi r^2 H$.
 - Hence, find the value of the expression when $\pi = \frac{22}{7}$, $r = 3$, $h = 10$ and $H = 12$.

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FACTORIZATION BY GROUPING

EXAMPLE

Factorize: $cx + cy + 2dx + 2dy$

SOLUTION

The terms cx and cy have c in common. Also, the terms $2dx$ and $2dy$ have $2d$ in common. Grouping in pairs this way,

$$\begin{aligned} cx + cy + 2dx + 2dy &= (cx + cy) + (2dx + 2dy) \\ &= c(x + y) + 2d(x + y) \\ &= (x + y)(c + 2d) \end{aligned}$$

EXAMPLE

Factorize: $3a - 6b + ax - 2bx$

SOLUTION

$$\begin{aligned} 3a - 6b + ax - 2bx &= 3(a - 2b) + x(a - 2b) \\ &= (3 + x)(a - 2b) \end{aligned}$$

EXAMPLE

Factorize: $2x^2 - 3x + 2x - 3$

SOLUTION

$$\begin{aligned} 2x^2 - 3x + 2x - 3 &= x(2x - 3) + 1(2x - 3) \\ &= (2x - 3)(x + 1) \end{aligned}$$

EXAMPLE

Factorize: $2am - 2m^2 - 3ab + 3bm$

SOLUTION

$$\begin{aligned} 2am - 2m^2 - 3ab + 3bm &= 2m(a - m) - 3b(a - m) \\ &= (a - m)(2m - 3b) \end{aligned}$$

EXAMPLE

Factorize: $cd - de + d^2 - ce$.

SOLUTION

d^2 and ce have no common factor, hence regroup the given terms.

Either

$$\begin{aligned} cd - de + d^2 - ce &= cd + d^2 - ce - de \\ &= d(c + d) - e(c + d) \\ &= (c + d)(d - e) \end{aligned}$$

Or

$$\begin{aligned} cd - de + d^2 - ce &= cd - ce + d^2 - de \\ &= c(d - e) + d(d - e) \end{aligned}$$

$$(c + d)(d - e)$$

EXAMPLE

Factorize: $2sru + 6tru - 4srv - 12trv$

SOLUTION

$2r$ is a factor of every term in the given expression.

$$\begin{aligned}2sru + 6tru - 4srv - 12trv &= 2r(su + 3tu - 2sv - 6tv) \\ &= 2r\{u(s + 3t) - 2v(s + 3t)\} \\ &= 2r(s + 3t)(u - 2v)\end{aligned}$$

EXERCISE

Factorize the following:

1. $cx - dx + 2cy - 2dy$
2. $2mh - 3nh - 3nk + 2mk$
3. $mn - 6pn + 3pm - 2n^2$
4. $2amu + 2anu - 2amv - 2anv$
5. $2d^2x + 4dx^2y - 3dy - 6xy^2$